In Section III analytical methods for extracting information about constitutive relations from measured wave profiles are discussed. In that section I propose a rigorous method for the reduction of pressure-time data that does not require the usual assumptions of steady state or equilibrium, isentropic flow.* Section IV presents a review of current knowledge of constitutive relations derived from plane-wave experiments.

A. Some Properties of Shock Waves

1. Jump Conditions

By "shock front" one means a compressive (usually) wave front which is steady. That is, a shock front transforms the mechanical and thermodynamic state of the material to one of higher stress, density, energy and mass velocity. The initial and final states are equilibrium states but the transition region, or shock front, necessarily involves non-equilibrium states. Because the transition region is steady, no mass, momentum, or energy accumulates within it and conservation laws can be applied to relate the initial and final states. These conservation laws, called the Rankine-Hugoniot jump conditions, can be written:[8]

$$V_1/V_o = 1 - [(u_1-u_o)/(U-u_o)] \tag{1}$$

$$P_1 - P_o = \rho_o(U-u_o)(u_1-u_o) \tag{2}$$

$$E_1 - E_o = [(P_1+P_o)/2](V_o - V_1) \tag{3}$$

In these equations V ($= \rho^{-1}$) is specific volume, P is the component of normal stress in the direction of shock propagation (not necessarily an equilibrium stress), E is specific internal energy, u is mass velocity, and U is the shock front velocity. Subscripts o refer to the state

*While this paper was in preparation I learned that Mr. Roger Williams had made the same discovery independently.[37]

ahead of the shock, while subscripts 1 refer to the shocked state.

It is readily shown that these equations imply that the entropy of the shocked state is higher than that of the initial state.[2,8] The locus of equilibrium states given by Eq. (3) is, for a given material in a given initial state, a unique function called the Hugoniot, or R-H curve, which therefore lies above the isentrope through the initial state. A complete equation of state for a material can be characterized by a family of R-H curves centered on different initial states.

Note that the components of normal stress parallel to the shock front do not enter the equations directly. They influence the shock only through the equation of state, which properly should be termed the equilibrium constitutive relation for one-dimensional compression.

The jump conditions apply not only to the equilibrium end states but throughout the transition region since each portion of the front is also steady. Equations (1) and (2) can be combined to give:

$$U - u_o = V_o \sqrt{(P_1 - P_o)/(V_o - V_1)} \qquad (4)$$

Since all parts of the wave travel with the same velocity, $U - u_o$, with respect to the undisturbed material, the locus of P, V, E states in the transition must lie on the straight line joining the initial and end states in the P-V plane. This line is called the Rayleigh line. (Figure 1.)

The difference between the Rayleigh line and the R-H curve at a given volume is approximately the non-equilibrium stress obtaining in the transition region and is primarily responsible for the entropy production.* If the material is treated as a viscous fluid, the steady-state shape of the shock front can be derived by relating the non-equilibrium stress to the stress rate or strain rate.[9,10]

2. Stability of Shock Waves

*The thermal part of this difference is normally negligible in solids.